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Variational time integrators for finite-dimensional thermo-elasto-dynamics without heat conduction

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SUMMARY

This paper focuses on the formulation of variational integrators (VIs) for finite-dimensional thermo-elastic systems without heat conduction. The dynamics of these systems happen to have a Hamiltonian structure, after thermal displacements are introduced. It is then possible to formulate integrators by taking advantage of standard methods in VI. The class of integrators we construct have some remarkable features: (a) they are symplectic, (b) they exactly conserve the entropy of the system, or in other words, they exactly satisfy the second law of the thermodynamics for reversible adiabatic processes, (c) they nearly exactly conserve the value of the energy for very long times and (d) they exactly conserve linear and angular momentum. We first describe how to adapt any VI for the classical mechanical systems to integrate adiabatic thermo-elastic ones, and then formulate three new types of integrators. The first class, based on a generalized trapezoidal rule, gives rise to two first order, explicit integrators, and a second order, implicit one. By composing then the two first-order integrators we construct a second order, explicit one. Finally, we formulate a fourth order, implicit integrator, which is a symplectic partitioned Runge–Kutta method. The performance of these new algorithms is showcased through numerical examples. Copyright © 2011 John Wiley & Sons, Ltd.

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KEY WORDS: time-stepping scheme; variational integrators; symplectic momentum-conserving methods; thermo-elasticity

1. INTRODUCTION

Early approaches toward the creation of time integrators for the dynamics of deformable bodies consisted in constructing a discretization of the momentum balance equations, without accounting for additional structure these equations might have had, see for example Chapter 9 of [1] or Chapters 9 and 10 of [2].

Pioneering contributions in the development of the so-called *conserving schemes* are for example [3, 4], among others. Conspicuous among these are the energy–momentum algorithms by Simo *et al.* [5, 6], which conserve energy as well as linear and angular momentum. These have been further extended to consider mechanical problems described by partial differential equations, such as encountered in the non-linear dynamics of solids [7], rods [8, 9] and shells [10]. The list of contributions in this area is long, with for example [11–15] among others.

Variational integrators (VIs) constitute a more recent approach toward the creation of structure-preserving integrators. The construction of VI is rooted in the formulation of a discrete analog to Hamilton's variational principle. These ideas were first developed in the context of integrable

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